FUNDAMENTAL EQUATIONS IN CREEP THEORY

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Development of modern technology requires solution of various applied problems about the behavior of structural materials with all of the complicating conditions of operation, as a rule accompanied by material creep. Central to this problem is use of creep theory. Recently versions of this theory have been suggested. In spite of the complexity of the majority of theories they are limited in nature and have a capacity to predict more or less material behavior within narrow changes in stresses and only some of the facts observed in experiments. There is entirely insufficient experimental work. It is impossible to find in the literature data about a single material embracing a broad selection of stresses, loading paths, and type of stressed state, and available experimental results are often supplied in a form convenient for confirming the conclusions at which the author has arrived.

It is desirable to assess the merits of one or another version of creep theory by comparing its predictions with experimental data not with smooth but with contrasting changes in the force and temperature conditions, for example such as stepwise loading with repeated steps, a change in the sign of stresses, a sharp change in the form of stressed state (torsion to tension and conversely), etc. One of the important experimental facts, i.e., anisotropy of strengthening with creep, was apparently first noted in [1].

It has been established by experiment that with the same value of stress intensity the creep process with pure torsion (tension) following prior creep with pure tension (torsion) accurately reproduces the creep process with torsion (tension) for a previously undeformed specimen (Fig. 1). Thus, prior creep in tension (torsion) does not affect subsequent creep in torsion (tension). Subsequently, other information was obtained about the anisotropic nature of strengthening, and currently the anisotropy of strengthening with creep is generally acknowledged.

Many authors have suggested different versions of creep theory with anisotropic strengthening whose characteristic feature is introduction of additional kinematic parameters (sometimes as strain components) or additional stresses. However, as a rule in all of these versions noted above experimental fact was ignored. Only work in [2] is an exception.

The independence of creep in torsion (tension) on prior creep in tension (torsion) may be explained by staying within the limits of normal strengthening theory with suitable selection of a strengthening parameter whose values in torsion and tension would be independent. This version of creep theory was suggested in [3] where as a measure of strengthening

$$\Omega = p_{ij}\sigma_{ij} \tag{1}$$

was taken (σ_{ij} and p_{ij} are creep stress and strain tensor components). Creep strain rate

$$\dot{p}_{ij} = h(\Omega)\partial f(\sigma_i,\ q)/\partial\sigma_{ij}. \tag{2}$$

Here q is a parameter making it possible to consider the dependence of creep rate on the type of stressed state; h and f are functions of these arguments;

$$p_i = \left(\frac{2}{3}\,p_{ij}p_{ij}\right)^{1/2},\quad \sigma_i = \left(\frac{3}{2}\,\bar{\sigma}_{ij}\bar{\sigma}_{ij}\right)^{1/2},\quad \sigma_0 = \sigma_{ii}/3,\ \bar{\sigma}_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0 \tag{3}$$

(δ_{ij} is Kronecker symbol)

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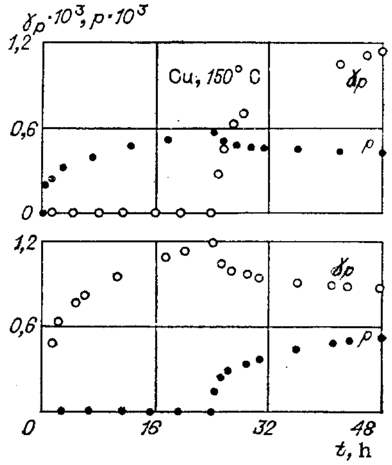

Fig. 1

For commercially pure copper q = 0 since the creep curve on coordinates p_i, t with fixed σ_i is a single curve for all forms of stressed state [4]. With proportional loading $\Omega = p_i\sigma_i$ and from (2) normal creep theory follows. It is evident that from relationships (1) and (2) there is independence of creep in tension (torsion) on preceding creep in torsion (tension).

Now we return to material behavior under creep conditions with reversal of the load. As was shown in [5], in creep experiments in torsion prior creep with stress of a single sign has a weakening effect on subsequent creep with stress of the opposite sign. Creep strain accumulated with reverse torsion markedly exceeds creep strain accumulated with direct torsion. A similar result was obtained later with reversal of both tension [6] and torsion [7-12]. Attempts to describe this fact analytically, which are undoubtedly a consequence of strengthening anisotropy with creep, have been attempted by a number of authors [2, 8-10, 13, 14].

It is easy to understand that with a sufficient number of load reversal cycles total creep strain p^+ accumulated at the end of a direct deformation half-cycle should be equal to the total creep strain p^- accumulated at the end of the reverse deformation half-cycle since the material is "indifferent" to with which half-cycle cyclic deformation commenced. So that this process is possible, creep strain accumulated per half-cycle of reverse deformation in absolute value should be greater than the creep strain accumulated per half-cycle of direct deformation in each of the initial cycles of load reversal until $|p^-| = |p^+|$. Results of experiments confirm this ([5-12]). However, according to the theory suggested considering load reversal the deformation process is as a rule established from the first cycle and $|p^+| > |p^-|$, i.e., the material should "remember" from which half-cycle load reversal commenced. Often only the first cycles or one of the first cycles are considered.

One of the versions of fundamental equations making it possible to predict the creep process with load reversal may be based on the suggestion of separating rules for deformation with direct and reverse loading similar for rules for active loading and unloading in plasticity theory (similar to the suggestion made in [14]). Furthermore, it is suggested that with a reduction in loads the creep process consists of strain accumulation with changes in values of stress and recovery [15].

By embodying these ideas in a strengthening hypothesis with a uniaxial stressed state, a fundamental relationship may be written as follows. With an active process

$$\dot{p}_a |p_a|^\alpha = f(\sigma)\sigma, \ \sigma d\sigma \geqslant 0, \ p = p_a. \tag{4}$$

After unloading stresses from σ_0 to σ_r without a change in the sign of the stress

$$\begin{gathered} \dot{p}_a |p_a|^\alpha = f(\sigma_r)\sigma_r, \ \sigma_0 d\sigma < 0, \ \sigma_0\sigma_r > 0, \\ \dot{p}_r |p_r|^\beta = A p_0{}^b |\sigma_0 - \sigma_r|^N, \ p = p_a - p_r, \end{gathered} \tag{5}$$

where p_r is recovery strain; p_0 is creep strain at the instant of the start of unloading; p is total creep strain.

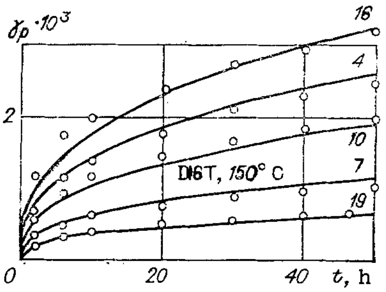

Fig. 2

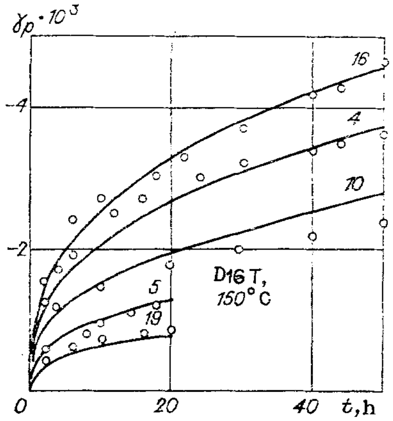

Fig. 3

After unloading stresses from σ_0 to σ_r with a change in the sign of the stress

$$\dot{p}_b|p_b|^\alpha = f(\sigma_r)\sigma_r,\ \sigma_0 d\sigma < 0,\ \sigma_0\sigma_r < 0,$$
$$\dot{p}_r|p_r|^\beta = Ap_0{}^b|\sigma_0 - \sigma_r|^N,\ p = p_0 + p_b - p_r \tag{6}$$

(α, β, b, A, N are material constants).

Creep strain p_b accumulated with changing stress sign and magnitude is not assumed to be dependent upon strain for the active process p_a. After a change in stress sign total creep strain p equals the sum of strain p_0 accumulated in the active process before unloading and strain p_b accumulated after unloading leaving out recovery strain p_r.

With pure shear in these relationships it is necessary to substitute p by $\gamma_p/\sqrt{3}$, and σ by $\sqrt{3}\tau$ (γ_p is creep shear strain, τ is tangential stress).

Presented in Fig. 2 are creep curves with direct torsion, and in Fig. 3 with reverse torsion; points are experimental data in [5], and lines are the results of calculation by relationships (6). Shown in Fig. 4 is a creep curve for alloy D16T with repeated reversal of the tangential stress obtained by relationships (6), and points are the results of experiments [5]. As can be seen, the creep process arrives at a steady regime in which $|p^+| = |p^-|$, which confirms the conclusion drawn above. The process is almost established after about 25 half-cycles. A similar picture was also observed with creep without strengthening [16]. The amplitude of plastic strain in this case increases with time, which is entirely natural with accelerated creep. By introducing a damage parameter it is easy to obtain the effect of growth of plastic strain amplitude with time.

The version of fundamental equations (4)-(6) with increasing stress conforms with normal strengthening theory. With a stepwise increase in strength the strength theory gives somewhat lower values of creep strain compared with experiments. This deficiency is typical for the majority of known creep theories. In almost all experimental work in which a stepwise increase in stress is considered it is noted that following an increase in stress creep strain increases markedly, and then it decreases approaching that corresponding to the new stress level.

A number of attempts have been made to try an analytical description of this effect [17, 18]. In [17] an additional parameter was introduced into the strengthening hypothesis which was the additional work for plastic creep strain. In [18] instead of creep strain the work of creep strain was taken as a measure of strengthening in the strengthening hypothesis. Subsequently these ideas were further developed and they were called an energy version of creep theory [19].

With a stepwise reduction in stress, as experimental data show, there are permissible versions of $\dot{p} \leqslant 0$. Relationships (6) make it possible to obtain this result.

Fundamental relationships (4)-(6) are easily generalized in the case of a complex stressed state. With an active process ($\sigma_{\beta\delta}d\sigma_{\beta\delta} > 0$)

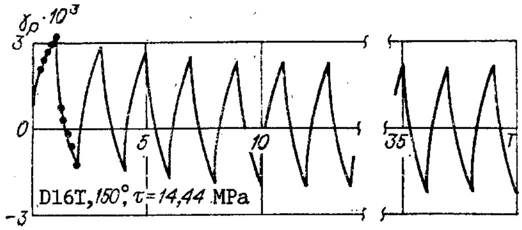

Fig. 4

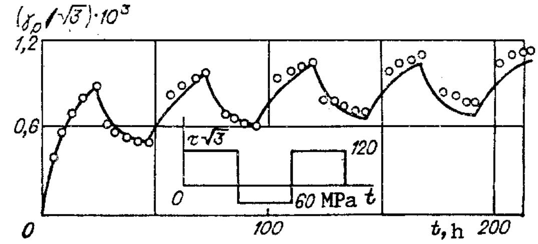

Fig. 5

$$\dot{p}_{\beta\delta a} = \frac{3}{2} a\Omega^{-\alpha}\sigma_i^{n-\alpha-1}\bar{\sigma}_{\beta\delta} \exp\left(C\int_0^t p_{ija} d\sigma_{ij} + B\frac{|T_3|}{\sigma_i^3}\right), \quad p_{\beta\delta} = p_{\beta\delta a}. \tag{7}$$

After decreasing stresses $\sigma_{\beta\delta}$ from $\sigma_{\beta\delta_0}$ to $\sigma_{\beta\delta_r}$ without changing the sign ($\sigma_{\beta\delta_0} d\sigma_{\beta\delta} < 0$, $\sigma_{\beta\delta_0}\sigma_{\beta\delta_r} > 0$)

$$\begin{gathered} \dot{p}_{\beta\delta a} = \frac{3}{2} a\Omega^{-\alpha}\sigma_{ir}^{n-\alpha-1}\bar{\sigma}_{\beta\delta r} \exp\left(C\int_0^t p_{ija} d\sigma_{ij} + B\frac{|T_3|}{\sigma_i^3}\right), \\ \dot{p}_{\beta\delta r} = \frac{3}{2} A p_i^b (\Omega')^{-\beta} (\sigma_i')^{N+\beta-1} \bar{\sigma}'_{\beta\delta} \exp\left[B\frac{|T_3'|}{(\sigma_i')^3}\right]. \end{gathered} \tag{8}$$

Total creep strain

$$p_{\beta\delta} = p_{\beta\delta a} - p_{\beta\delta r}. \tag{9}$$

With $d\sigma_i < 0$ it is necessary to assume that C = 0. After reducing stresses $\sigma_{\beta\delta}$ from $\sigma_{\beta\delta_0}$ to $\sigma_{\beta\delta_r}$ with a change in sign ($\sigma_{\beta\delta} d\sigma_{\beta\delta} < 0$, $\sigma_{\beta\delta_0}\sigma_{\beta\delta_r} < 0$)

$$\begin{gathered} \dot{p}_{\beta\delta b} = \frac{3}{2} \Omega_b^{-\alpha} a \sigma_{ir}^{n-\alpha-1}\bar{\sigma}_{\beta\delta r} \exp\left(B\frac{|T_3|}{\sigma_{ir}^3}\right), \\ \dot{p}_{\beta\delta r} = \frac{3}{2} A p_i^b (\Omega')^{-\beta} (\sigma_i')^{N+\beta-1} |\bar{\sigma}'_{\beta\delta}| \exp\left[B\frac{|T_3'|}{(\sigma_i')^3}\right]. \end{gathered} \tag{10}$$

Total creep strain

$$p_{\beta\delta} = p_{\beta\delta a} + p_{\beta\delta b} - p_{\beta\delta r}. \tag{11}$$

Here $\sigma_{ij}' = \sigma_{ij0} - \sigma_{ijr}$; $\Omega' = p_{ijr}\sigma_{ij}'$; $\Omega_b = p_{ijb}\sigma_{ijr}$; $T_3 = 9\bar{\sigma}_{ij}\bar{\sigma}_{jk}\bar{\sigma}_{ki}$; a, α, n, C, B, b, A, N, β are material constants at a given temperature. The term with the third invariant of the stress deviator T_3 takes account of the dependence of creep rate on the type of stressed state. If the creep curve $p_i(t)$ does not depend on the type of stressed state with the same stress intensity σ_i, then in relationships (7), (8), and (10) it is necessary to take B = 0. The term with parameter C takes account of the increase in creep rate with a stepwise increase in stress.

The suggested model (7)-(11) is a good quantitative description of all experimental results provided in [1-5], including the fact of independence of the next creep in torsion (tension) on the previous creep in tension (torsion), load reversal, and also the recovery of prior creep strain in tension (torsion) in subsequent creep in torsion (tension) observed in [4] (see Fig. 1).

Among other experimental works of greatest interest is that in [10] which contains various test programs. However, it should be noted that in these experiments test duplication is almost absent, and the scatter observed for data was quite large. In tests with the same stress intensity σ_i the authors obtained nonconformity of creep curves on coordinates p_i, t with torsion and tension. They put this effect down to anisotropy and the theory suggested by them is not described, but it is observed for isotropic materials in the original condition and consequently it is impossible to put it down to anisotropy. Description of this effect in model (7)-(11) is accomplished by introducing the third invariant of the stress deviator T_3.

The model suggested (7)-(11) is in good agreement with experimental data [10]. Given as an example in Fig. 5 is one of the versions of these experiments where the line is the result of calculation, and points are for an experiment.

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